SMO Algorithm

Initialize $\alpha_i \leftarrow 0$ and $b \leftarrow 0$
Let $f_a(\mathbf{x}) = b + \Sigma_{i=1}^{m} y_i \alpha_i k(\mathbf{x}, \mathbf{x}_i)$
Let τ be the tolerance.
Loop

Find two exs. $(\mathbf{x}_p, y_p)$ and $(\mathbf{x}_q, y_q)$ such that:

$$(f_a(\mathbf{x}_p) - y_p + \tau < f_a(\mathbf{x}_q) - y_q - \tau) \wedge$$
$$((\alpha_p < C \wedge y_p = 1) \vee (\alpha_p > 0 \wedge y_p = -1)) \wedge$$
$$((\alpha_q > 0 \wedge y_q = 1) \vee (\alpha_q < C \wedge y_q = -1))$$

exit loop if no such examples can be found

$$\eta \leftarrow \frac{(f_a(\mathbf{x}_q) - y_q) - (f_a(\mathbf{x}_p) - y_p)}{k(\mathbf{x}_p, \mathbf{x}_p) - 2k(\mathbf{x}_p, \mathbf{x}_q) + k(\mathbf{x}_q, \mathbf{x}_q)}$$

if needed, reduce η so that:

$$(0 \leq \alpha_p + y_p \eta \leq C) \wedge$$
$$(0 \leq \alpha_q - y_q \eta \leq C)$$

$\alpha_p \leftarrow \alpha_p + y_p \eta$
$\alpha_q \leftarrow \alpha_q - y_q \eta$

End Loop
$b_1 \leftarrow \max \{f_a(x_i) \mid y_i = 1 \wedge \alpha_i > 0\}$
$b_{-1} \leftarrow \min \{f_a(x_i) \mid y_i = -1 \wedge \alpha_i > 0\}$
$b \leftarrow -(b_1 + b_{-1})/2$

SMO Explanation

SMO means "sequential minimal optimization".

The conditions on $\mathbf{x}_p$ and $\mathbf{x}_q$ mean:

1. Relative to each other, $\mathbf{x}_p$ is below its margin boundary, and $\mathbf{x}_q$ is above its boundary.
2. The tolerance is a tradeoff between accuracy and efficiency.
3. α_p can be changed to increase $f_a(\mathbf{x}_p)$.
4. α_q can be changed to decrease $f_a(\mathbf{x}_q)$.

To maintain $\sum_{i=1}^{m} \alpha_i y_i = 0$, an increase in $\alpha_p y_p$ is matched by a decrease in $\alpha_q y_q$.

η optimizes the objective function.

$0 \le \alpha_i \le C$ must be maintained.

One way to set the bias weight b is shown.

Many details to make this more efficient and stable have been omitted.

J. Platt, Fast training of support vector machines using sequential minimal optimization, in *Advances in Kernel Methods – Support Vector Learning* (B. Scholkopf, C.J.C. Burges, and A.J. Smola, eds.), MIT Press, 185-208, 1999.